Descubre cómo mis alumnos resolvieron problemas reales de examen 🧐

Aquí pongo la solución de 3 problemas que vinieron en exámenes reales pasados (examen tipo ETS, para quienes estudian en el IPN). Estos problemas los resolvimos durante la clase, así que puedes estar seguro de que las soluciones son correctas.

Les doy mucho crédito a mis alumnos por su esfuerzo, ya que ellos mismos las hicieron, yo solo me encargué de escanearlas para que tú las puedas revisar.

¡Échale un ojo a las soluciones y úsalo como referencia para tu estudio! Siempre es útil ver cómo otros estudiantes resuelven problemas tipo examen. ¡Espero que te ayuden a entender mejor la materia y te sirvan de base para prepararte!

$$(1+x^2)y' + xy = 0$$

$$(1+x^2)\frac{dy}{dx} + xy = 0$$

$$(1+x^2)\,dy = -xy\,dx$$

$$1+x^2\,dy = -xy\,dx$$

$$\frac{dy}{y} = \frac{-x}{1+x^2}\,dx$$

$$\int \frac{dy}{y} = \int \frac{-x}{1+x^2}$$

$$\ln y = \int \frac{-x}{1+x^2} \rightarrow \text{cambio de variabl}$$

$$\ln y = -\frac{1}{2}\ln(1+x^2) \underline{+c}$$

$$(1+x^2)y' + xy = 0$$

$$(1+x^2)\frac{dy}{dx} + xy = 0$$

$$(1+x^2)\,dy = -xy\,dx$$

$$\frac{dy}{y} = -\frac{x}{1+x^2}\,dx$$

$$\int \frac{dy}{y} = -\int \frac{x}{1+x^2}\,dx$$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\ln(y) = -\int \frac{du}{2u}$$

$$\ln(y) = -\frac{1}{2}\int \frac{du}{u}$$

$$\ln(y) = -\frac{1}{2}\ln(u)$$

$$\ln(y) = -\frac{1}{2}\ln(1+x^2) + c$$

Ejercicio 4.

$$(4y + yx^2)\,dy - (2x + xy^2)\,dx = 0$$

$$(4y + yx^2)\,dy = (2x + xy^2)\,dx$$

$$y(4 + x^2)\,dy = x(2 + y^2)\,dx$$

$u = 4 + x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$$\frac{y\,dy}{2 + y^2} = \frac{x}{4 + x^2}\,dx$$

$$\int \frac{x}{4+x^2}\,dx$$

$$\int \frac{y}{2+y^2}\,dy$$

$$\frac{1}{2}\int \frac{du}{u} = \frac{\ln}{2}$$

$u = 2 + y^2$

$\frac{du}{dy} = 2y$

$\frac{du}{2} = y\,dy$

$$\frac{1}{2}\int \frac{du}{u}$$

$$\frac{\ln(2+y^2)}{2} = \frac{\ln(4+x^2)}{2} + C$$

$(4y + yx^2)\,dy - (2x + xy^2)\,dx = 0$

$(4y + yx^2)\,dy = (2x + xy^2)\,dx$

$y(4 + x^2)\,dy = x(2 + y^2)\,dx$

$$\frac{y}{2+y^2}\,dy = \frac{x}{4+x^2}\,dx$$

$$\int \frac{y}{2+y^2}\,dy = \int \frac{x}{4+x^2}\,dx$$

$u = 2 + y^2$ $\quad\quad$ $u = 4 + x^2$

$du = 2y\,dy$ $\quad\quad$ $du = 2x\,dx$

$\frac{du}{2} = y\,dy$ $\quad\quad$ $\frac{du}{2} = x\,dx$

$$\int \frac{du}{2u} = \int \frac{du}{2u}$$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\int \frac{du}{u}$$

$$\frac{1}{2}\ln(u) = \frac{1}{2}\ln(u)$$

$$\frac{1}{2}\ln(2+y^2) = \frac{1}{2}\ln(4+x^2) + C$$

$$y' = \sqrt{x^2y^2 - y^2}$$

$$dy = \sqrt{x^2y^2 - y^2}\, dx$$

$$dy = \sqrt{y^2(x^2-1)}\, dx$$

$$dy = \sqrt{y^2}\sqrt{x^2-1}\, dx$$

$$\frac{dy}{\sqrt{y^2}} = \sqrt{x^2-1}\, dx$$

$$u^2 = x^2 \qquad a^2 = 1$$

$$u = x \qquad a = 1$$

$$\int \frac{dy}{y} = \int \sqrt{x^2-1}\, dx \rightarrow \frac{1}{2} u \sqrt{u^2-a^2} - \frac{1}{2} a^2 \ln\left|u + \sqrt{u^2-a^2}\right.$$

$$\ln(y) = \frac{1}{2} x \sqrt{x^2-1} - \frac{1}{2}(1) \ln\left|x + \sqrt{x^2-1}\right| + 1$$

$$\ln(y) = \frac{1}{2} x \sqrt{x^2-1} - \frac{1}{2}(1) \ln\left|x + \sqrt{x^2-1}\right| + 1 + C$$

⑤ $(2t+5)r' + (t^2-5)r = 0$ $\qquad r' = \frac{dr}{dt}$

$(2t+5)\frac{dr}{dt} + (t^2-5)r = 0$

$(2t+5)\frac{dr}{dt} = -(t^2-5)r$

$\frac{dr}{r} = \frac{-(t^2-5)}{2t+5}dt \longrightarrow \int\frac{dr}{r} = -\int\frac{t^2-5}{2t+5}dt$

$\rightarrow \ln r = -\int \left(\frac{1}{2}t - \frac{5}{4} + \frac{\frac{5}{4}}{2t+5}\right) dt$

$\rightarrow \ln r = -\left[\int\frac{1}{2}t\,dt - \int\frac{5}{4}dt + \int\frac{\frac{5}{4}}{2t+5}dt\right]$

$\rightarrow \ln r = -\frac{t^2}{4} + \frac{5}{4}t - \frac{5}{8}\ln|2t+5| + c$

$$(2t-5)r' + (t^2-5)r = 0$$

$$(2t-5)\frac{dr}{dt} = -(t^2-5)r$$

$$(2t-5)dr = -(t^2-5)r\,dt$$

$$\int \frac{dr}{r} \ (1) = -\int \frac{t^2-5}{2t-5}\,dt \ (2)$$

(2)

$$2t-5 \,\overline{)\, t^2-5} \longrightarrow 2t-5 \,\overline{)\, t^2+0t-5}$$

$$\frac{1}{2}t + \frac{5}{4}$$

$$-t^2 + \frac{5}{2}t$$

$$0 + \frac{5}{2}t - 5$$

$$-\frac{5}{2}t$$

$$0 + \frac{25}{4}$$

$$\frac{5}{4}$$

(1)

$$\boxed{\frac{t^2}{2t-\cancel{5}} = \frac{t}{2}}$$

(2)

$$\boxed{\frac{\frac{5}{2}\cancel{t}}{2\cancel{t}} = \frac{5}{4}}$$

$$-\left(\int \frac{1}{2}t\,dt + \int \frac{5}{4}\,dt + \frac{5}{4}\int \frac{dt}{2t-5}\right.$$

$$-\int\left(\frac{1}{2}\cdot\frac{t^2}{2}\right) + \frac{5}{4}t + \frac{5}{4}\left(\frac{1}{2}\int \frac{du}{u} = \ln\right.$$

$$-\left(\frac{t^2}{4} + \frac{5}{4}t + \frac{5}{4}\left(\frac{1}{2}\ln u\right)\right)$$

$$u = 2t-5$$

$$\frac{du}{dt} = 2$$

$$\frac{du}{2} = dt$$

$$-\left(\frac{t^2}{4} + \frac{5}{4}t + \frac{5}{8}\ln(2t-5)\right) \longrightarrow \text{Resultado final}$$

$$\ln r = -\frac{t^2}{4} - \frac{5}{4}t + \frac{5}{8}\ln(2t-5$$